Test 3A - MTH 1410

Dr. Graham-Squire, Spring 2013

Name: ______________________________

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Unless otherwise stated, you should <u>use calculus</u> to justify your answers (in other words, just looking at a graph is NOT enough of a reason).
3. Clearly indicate your answer by putting a box around it.
4. Cell phones and computers are <u>not</u> allowed on the test. Calculators are allowed on the first 6 questions, but are <u>not</u> allowed on the last 2 questions of this test.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. Make sure you sign the pledge.
7. Number of questions = 8. Total Points = 80.

1. (12 points) Find the following limits. Make sure to use correct notation.

   (a) $\displaystyle\lim_{x\to\infty} \frac{\ln x}{x}$

   (b) $\displaystyle\lim_{x\to 0^+} \frac{\ln x}{x}$

   (c) $\displaystyle\lim_{x\to\infty} 2x \sin\left(\frac{1}{x}\right)$

2. (10 points) Helen of Troy is standing atop a vertical cliff rising one mile above the ocean. She is watching as one of the thousand ships launched for her face sails away from her. If the boat is 2 miles from the base of the cliff and is moving on the water away from her at a speed of 10 mi/hr, how fast is the (diagonal) distance between Helen and the boat increasing? Round your answer to the nearest 0.01 miles/hour.

3. (10 points) Find the absolute maximum and absolute minimum of the function

$$f(x) = e^{x^3 - 5x^2 + 3x}$$

on the interval [0,4.5]. Round your answers to the nearest 0.01.

4. (10 points) Use differentials to approximate the change in the surface area of a sphere when the radius is increased from 50 cm to 50.2 cm. Round your answer to the nearest 0.01 cm. The surface area of a sphere is given by $S(r) = 4\pi r^2$.

5. (10 points) Consider the function $f(x) = \dfrac{\ln x}{x}$. It has the derivatives:

$f'(x) = \dfrac{1 - \ln x}{x^3}$ and $f''(x) = \dfrac{2\ln x - 1}{x^3}$.

Find the following, and make sure to show your work:

(a) The x-value for all local maximums, if any exist.

(b) The interval(s) where the function is decreasing.

(c) The interval(s) where the function is concave down.

(d) The x-value(s) of the inflection point(s), if any exist.

6. (10 points) Postal regulations have the following stipulation for rectangular boxes that have a square bottom: the sum of the length of the base and the length of the height cannot exceed 10 feet. Find the <u>maximum</u> volume for such a box. Round your answer to the nearest 0.01 ft^3.

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7. (10 points) Use logarithmic differentiation to find the derivative of $y = (\sin x)^x$.

8. (8 points) An astronaut on the moon throws a ball vertically upward. The height of the ball is given by the function

$$h(t) = 24t - 4t^2$$

where the height is given in feet, $t = 0$ is when the ball is released, and t is measured in seconds. Use calculus to solve an explain the following questions.

(a) When does the ball reach its maximum point? Use calculus to explain how you know it is a maximum.

(b) What is the highest point the ball reaches?

Extra Credit(up to 3 points) Write either a 1 or a 3 into the space below to request how many points you want for extra credit. If you put a 1 you are guaranteed 1 point. If you put a 3 and less than half the class also puts a 3, then you get 3 points. If more than half the class puts a 3, you get zero.